PHYSICS DEPARTMENT
PRINCETON UNIVERSITY

GRADUATE GENERAL EXAMINATION

Monday, January 8, 2001 - 9:00 am - 12:00 noon

Part III.

This part of the General Examination poses three questions on Condensed Matter Physics and three on Elementary Particles and Nuclear Physics. Answer three questions, at least one from each section.

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

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Part III. Section A. Condensed Matter Physics

1. Consider the following data for the heat capacity of potassium at low temperatures:

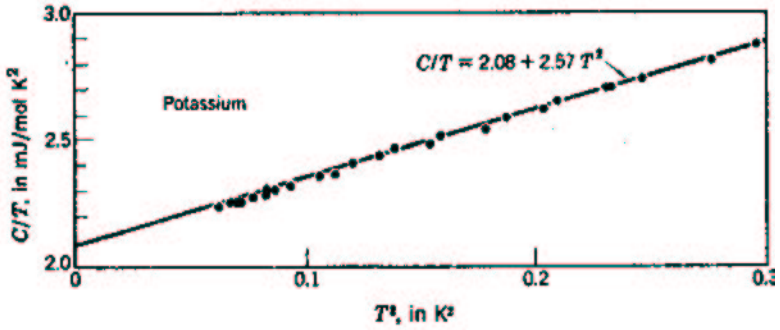

Figure: Experimental heat capacity values *vs.* temperature for potassium, plotted as C/T versus T^2. The fit has the functional form $C/T = 2.08 + 2.57T^2$ mJ/mol K^2. [After W. H. Lien and N. E. Phillips, Phys. Rev. **133**, A1370 (1964).]

a) Identify the physical origins of the two terms in the fit to the data.

b) Calculate the contribution of the conduction electrons to the bulk modulus of a metal in the free electron approximation.

c) Using the data on the heat capacity provided above, estimate the contribution of the conduction electrons to the zero temperature bulk modulus of potassium.

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Part III. Section A. Condensed Matter Physics (continued)

2. The ground state electronic wavefunction of the hydrogen atom is $\psi(r) = e^{-r/a_0}/\sqrt{\pi a_0^3}$ where $a_0 = 0.529 \times 10^{-8}$ cm is the Bohr radius.

   a) Using this wavefunction, and ignoring the interaction between the electrons, calculate the (electronic) molar magnetic susceptibility of helium at temperature $T = 0$. Is the susceptibility diamagnetic or paramagnetic?

   b) Lithium is next in the periodic table. What are the additional contributions to its magnetic susceptibility at $T = 0$ in its solid phase? Estimate the order of magnitude of these additional contributions and include their signs in your answer. (Each lithium atom occupies a sphere of radius approximately $3a_0$ in the solid.)

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Part III. Section A. Condensed Matter Physics (continued)

3. Aluminum is a superconductor with a transition temperature $T_c \simeq 1.2$ K, coherence length $\xi = 1\ \mu$m and penetration depth $\lambda = 160$ Å (16 nm). The Fermi energy of Al is 12 eV, corresponding to a conduction electron density $n \sim 18 \times 10^{22}/\text{cm}^3$, and its superconducting gap Δ at zero temperature is about 2K.

   a) A spherical sample of Al of radius r is placed, at zero temperature, in a magnetic field $B < B_c$ with a gradient $|\nabla B|$ (B_c is the critical field). Estimate the force on it. What is the direction of the force? What is the force for a sample with $r = 1$ mm, $B = 10$ G and $|\nabla B| = 1$ G/cm? What happens to this force as B is increased to 1000 G but the gradient is kept constant (justify your answer)?

   b) Imagine that a Dirac magnetic monopole and anti-monopole of unit strength (*i.e.*, their integrated magnetic fluxes are $\pm\phi_0$ with $\phi_0 = hc/e$) separated by a distance $R = 1$ cm are placed in a practically infinite piece of Al. Estimate the force between the monopole/anti-monopole pair.

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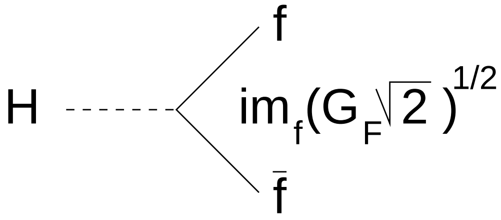
Part III. Section B. Elementary Particles and Nuclear Physics

1. There are reports of a possible signal for a Higgs boson of mass $M_H \approx 114$ GeV from the recently defunct LEP collider. The basic vertex for Higgs coupling to a fermion-antifermion pair is $im_f(G_F\sqrt{2})^{1/2}$, where m_f is the mass of the fermion:



a) Estimate the decay rate (width) for $H \to f\bar{f}$ for $M_H \gg m_f$.

b) For a 114-GeV Higgs, what is the dominant decay mode? What is the width in MeV in this case?

c) LEP is an e^+e^- collider. If the machine ran "on resonance" for direct production of this Higgs at a luminosity of $\mathcal{L} = 10^{32}$ cm^{-2}s^{-1}, how many events of the type $e^+e^- \to H \to f\bar{f}$, with $f\bar{f}$ the dominant decay mode, would be produced in one year?

d) The process of part c) is not the most advantageous one due to the small coupling of the Higgs to electrons. Draw a Feynman diagram of a process that could give a higher yield of Higgs production if LEP were to run at a center of mass energy above 205 GeV.

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Part III. Section B. Elementary Particles and Nuclear Physics (continued)

2. The prototype weak decay is nuclear β-decay, e.g., the decay of a free or bound neutron:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

a) Draw the leading Feynman diagram for neutron decay in terms of the quark constituents.

b) Using Fermi's Golden Rule and assuming a constant matrix element, derive an expression for the shape of the β decay electron energy spectrum. Determine the dependence of the β-decay rate on the Fermi coupling G and on Δm, the mass difference between neutron and proton. (You may make the approximation that the electron is extremely relativistic.)

c) The ground states of ${}^{34}_{17}\mathrm{Cl}$ and ${}^{34}_{16}\mathrm{S}$ have $J^P = 0^+$ and belong to an $I = 1$ isospin multiplet. The decay

$${}^{34}_{17}\mathrm{Cl} \rightarrow {}^{34}_{16}\mathrm{S} + e^+ + \nu_e$$

has a lifetime of 2.3 s and maximum positron energy of 4.5 MeV. Estimate the branching ratio for the decay

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e \ .$$

The lifetime and mass of π^+ and π^0 are 26 ns, 139.6 MeV, and 8×10^{-17} s, 135.0 MeV, respectively.

3. Our Milky Way has a nearly spherical halo of dark matter that comprises most of its mass. The dark matter is thought to consist of stable, neutral elementary particles. If the particles interact only gravitationally, then simulations suggest that their density should rise sharply as one approaches the galactic center. However, recent astronomical observations suggest that this is not the case. The dark matter density is nearly constant (about 0.4 GeV/cm^3) inside a radius of 8 kpc, about the distance of the sun from the galactic center (1 kiloparsec $= 3 \times 10^{21}$ cm). Within this radius, the particles travel at speeds of about 300 km/s.

A proposed explanation is that the dark matter particles may scatter elastically off one another. The scattering insures that they cannot pile up in the center of the galaxy and they are more evenly distributed within the galactic core. A few interactions per particle over the life time of the universe (15 billion years) suffices to alter their distribution.

a) If the dark matter particle has mass m, estimate the lower bound on the elastic scattering cross section required to alter their distribution.

b) Compare the scattering cross section of part a) to that for "WIMPS", hypothetical weakly interacting massive particles with $m \approx 100$ GeV.

This problem is continued on the next page.

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Part III. Section B. Elementary Particles and Nuclear Physics (continued)

Problem 3. (continued)

Suppose the dark matter particles scatter elastically from baryons with the same cross section with which they scatter from themselves. Then, existing dark matter detectors on the surface of the Earth or deep underground could, in principle, detect them. However, the particles would first have to penetrate the atmosphere. Assume that the atmosphere consists mostly of nitrogen.

c) Using the lower bound for the cross section found in part a) and assuming $m = 100$ GeV, estimate the interaction length (mean free path) in cm due to scattering in the atmosphere.

d) If the particle scatters from a nucleus in the detector, the recoil energy can be detected provided it exceeds 10 eV. Estimate the energy of the particle when it reaches a detector at sea level. Will the particle be detected?