PHYSICS DEPARTMENT
PRINCETON UNIVERSITY

GRADUATE GENERAL EXAMINATION

Tuesday, May 11, 1999 - 9:00 am - 12:00 noon

Part IV.

This part of the General Examination poses SIX questions, TWO on Relativity, and FOUR on General and Atomic Physics. You must do ONE relativity problem and TWO General and Atomic questions.

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

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Part IV. Section A. Relativity

1. A region of space has metric,

$$ds^2 = f(x)dt^2 - dx^2 - dy^2 - dz^2.$$

Assume $f(x) = 1$ for $x < 0$ and $f(x) = \psi$ for $x > a$ (where $\psi \neq 1$ is a positive constant). Let K be the inertial reference frame on the left ($x < 0$) with coordinates (t, x, y, z). Let K' be the inertial reference frame on the right (where $x > a$) with time defined by $t' = \psi^{1/2}t$ and $x' = x$, $y' = y$ and $z' = z$. You might wish to recall the equation for the curvature,

$$R_{ab} = \partial_c \Gamma^c{}_{ab} - \partial_a \Gamma^c{}_{cb} + \Gamma^c{}_{ab}\Gamma^d{}_{cd} - \Gamma^c{}_{db}\Gamma^d{}_{ca}, \qquad \Gamma^c{}_{ab} = \frac{1}{2}g^{cd}\left\{\partial_a g_{bf} + \partial_b g_{ad} - \partial_d g_{ab}\right\}.$$

a) A photon of angular frequency ω in K enters the region on the left. What will be the frequency of the photon in K' when it emerges on the right?

b) A particle enters the region from the left ($x < 0$) with velocity $v > 0$ in K. For what values of v will the particle emerge on the other side ($x > a$)? If it does, find the velocity of the particle in K'.

c) Assuming the cosmological constant is zero, prove that no physical matter distribution can create the metric in the question.

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Part IV. Section A. Relativity (continued)

2. A pulsar is a rotating star from which one can detect a burst of radio waves once per rotation. The arrival times of these bursts can be measured with very high precision. Consider a binary system composed of two $1.4 M_\odot \approx 2.8 \times 10^{30}$kg neutron stars in a circular orbit separated by 5 light-seconds. The orbital axis is tilted 60° relative to the line-of-sight to earth. The time of flight of a burst of radiation from the pulsar to the earth differs slightly from the value one would calculate in the Newtonian limit. Calculate the *difference* in this relativistic modification between moments when the pulsar is farthest from earth and when it is closest to earth. Use the value $G = 6.67 \times 10^{-11}\text{Nm}^2/\text{kg}^2$ for Newton's constant. You might also wish to recall the Schwarzschild solution:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 d\Omega^2.$$

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Part IV. Section B. General and Atomic Physics

1. At the front of a room is a bottle containing the questionable drink called Orbitz, which has suspended blobs in it. Don't drink the Orbitz! Keep an eye on the blobs as you do the other problems.

   a) Let us suppose the density ρ of the yellow blobs is 1.01 grams/cm^3. Estimate the settling times of the blobs from top to bottom if only water was in the bottle.

   b) Slowly twist the bottle and suddenly stop the bottle. Watch the motion of the blobs. Are the blobs suspended in a pure liquid, based on your observation?

   c) Estimate the Young's modulus E of the matrix the blobs are suspended in, based on your observations.

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Part IV. Section B. General and Atomic Physics (continued)

2. Michelson in 1920 measured the diameter of the star Betelgeuse by using two mirrors and looking at change in the interference pattern of the light from the two mirrors.

Consider two (mutually incoherent) point sources which are separated by a distance ρ, and are a distance R from a 2-slit filter. The 2 slits are separated by a distance d and have widths w much less than the wavelength of the light λ being observed.

a) Let the fringe visibility F be defined as $F = (I_{max} - I_{min})/(I_{max} + I_{min})$. What is F for $\rho = 0$? Find an expression for the mirror separation d in terms of λ, R and ρ where F is equal to 0. This distance is called the transverse coherence length γ.

b) What is the approximate transverse coherence length γ for orange light for a star which is 10 light years away and whose diameter is about the same as the sun's?

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Part IV. Section B. General and Atomic Physics (continued)

3. A recent colloquium discussed how to slow the group velocity $dn(\omega)/dk$ of light to 17 m/sec. This problem is a simplified version of that experiment. One way to model atomic transitions is via a damped harmonic oscillator. Let the electron of mass m and charge e move in a harmonic potential with undamped resonant frequency ω_o and with a damping constant γ. Suppose a plane electromagnetic wave of $E_o exp[i(kz - \omega t)]$ is incident on such a system, containing N atoms/volume.

a) Find the steady state complex polarizability of the atoms $\alpha(\omega)$.

b) Derive an expression for the complex index of refraction $n(\omega) = \sqrt{\epsilon}$, where ϵ is the dielectric constant of the gas. You can assume that the gas is dilute. Sketch the real and imaginary values of $n(\omega)$ near resonance, and indicate in the figure the frequency ω_s where the group velocity is at a minimum.

c) Suppose that there are two closely spaced transitions ω_1 and ω_2, $\delta\omega = \omega_1 - \omega_2 << \omega_1$. Hau et al. effectively inverted the population of transition 2 making $\gamma_2 = -\gamma_1$, and passed light through the sample at a frequency mid-way between ω_1 and ω_2. What problem did this solve?

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Part IV. Section B. General and Atomic Physics (continued)

4. At the front of the room are some compact discs (CD). You will also find laser pointers and a rulers.

a) CDs use a 16 binary bit encoding scheme for sound level. Your mission is to make some simple measurements and figure out how long a CD can play. Of course, you probably know the answer. You are supposed to derive a number from simple physics. You might want to know that audio frequencies go up to 40 kHz.

b) Sound levels β are measured by the decibel log scale (dB): $\beta = 10log(I/I_o$, where $I_o = 10^{-12}$ watts/m^2. What is the amplitude of the air molecule displacement A for a 0dB sound at 1 kHz, the softest sound you can hear? The density of air is about 1/1000 that of water.