PHYSICS DEPARTMENT
PRINCETON UNIVERSITY

GRADUATE PRELIMINARY EXAMINATION

Friday, May 4, 2001 — 9:00 am – 12:00 noon

Part II.

Answer two out of the three questions in Section A (Quantum Mechanics) and two out of the three questions in Section B (Thermodynamics and Statistical Mechanics).

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

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Part II. Section A. Quantum Mechanics

1. A spin $\frac{1}{2}$ particle with magnetic moment μ is fixed to a point in space. Let $|\uparrow\rangle$ and $|\downarrow\rangle$ denote the states with $S_z = \frac{1}{2}$ and $S_z = -\frac{1}{2}$. We turn on a constant magnetic field with magnitude B_0 and the direction is given by:

$$\vec{B} = B_0(\hat{x}\sin\theta\,\cos\phi + \hat{y}\sin\theta\,\sin\phi + \hat{z}\cos\theta).$$

Here θ and ϕ are constant angles.

a) Find the ground state. Denote it by $|\theta,\phi\rangle$.

Now we make ϕ change slowly with time as $\phi = \omega t$.

b) In the adiabatic limit that ω is very small, the wavefunction can be approximated as

$$|\,t\,\rangle \sim e^{i\varphi(t)}|\theta,\omega t\rangle.$$

Here $|\theta,\omega t\rangle$ is the state you found above. Find $\varphi(t)$. ($\varphi(\frac{2\pi}{\omega})$ is called the *Berry phase.*)

Suppose that at time $t = 0$ the particle is in the ground state $|\theta,0\rangle$. Now we turn on the magnetic field for a whole cycle until time $t = \frac{2\pi}{\omega}$. At the end of the cycle we keep the magnetic field at the constant final value $\vec{B} = B_0(\hat{x}\sin\theta + \hat{z}\cos\theta)$.

c) Find the probability, to leading order in ω, that at the end of the cycle the particle will be in the excited state.

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Part II. Section A. Quantum Mechanics (continued)

2. Consider two hydrogen atoms with a fixed distance r between their nuclei that is large compared to the size of the atoms. Treat the Coulomb interaction as instantaneous (no retardation), and neglect the interactions between the spins.

a) The ground state energy of this pair of atoms depends on r as $C_0 + A_0 r^{-\delta_0} + \cdots$, where C_0, A_0 are constants. Find δ_0.

b) Give an order of magnitude estimate for A_0 and give a general argument why A_0 should be negative.

c) Now consider the first excited state of the system (keeping the distance r between the nuclei fixed and large). The energy depends on r as $C_1 + A_1 r^{-\delta_1} + \cdots$. Find δ_1.

d) Estimate at what distance (between the atoms) you will have to take into account the retardation effects in electro-magnetism.

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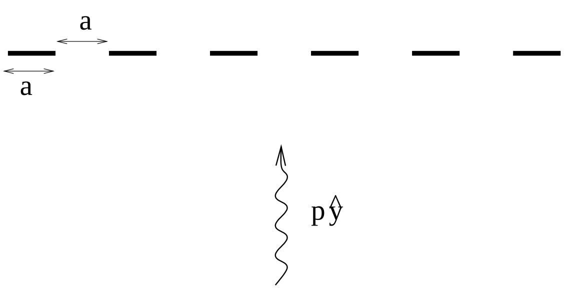
Part II. Section A. Quantum Mechanics (continued)

3. A spinless particle of mass m is confined to move in two dimensions. On the $\hat{x}$ axis we place a grid that can be modeled by the following potential:

$$V(x,y) = \lambda\delta(y) \qquad 2na \leq x \leq (2n+1)a$$

$$= 0 \qquad (2n+1)a < x < (2n+2)a$$

The particle is approaching the grid from below with momentum $\vec{p} = p\hat{y}$.



a) Using the Born approximation, find an expression for the probability for transmission.

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Part II. Section B. Thermodynamics and Statistical Mechanics

1. The problem addressed here is how one can measure Planck's constant and/or Avogadro's number by using principles of statistical mechanics and thermodynamics. The first part should be familiar; the second possibly less so. (Both types of measurements, corresponding to parts a) and b) below, have actually been done!)

   a) Assume that you know how to measure light frequency, temperature, and energy. Describe a Gedanken experiment for how you can measure Planck's constant h and Avogadro's number A. Give a formula relating both constants to measured quantities. (You can assume that the gas constant R has been measured as well.)

   b) Now instead of light frequency, suppose you can measure heat input at constant volume. Assuming the third law of thermodynamics (what, exactly, does it say?) and knowledge of A, how can you measure h by purely thermodynamic means? Give a formula for h in terms of your proposed measurement.

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Part II. Section B. Thermodynamics and Statistical Mechanics (continued)

2. A certain polymer is a chain of N molecules that touch each other. Each molecule can exist in two states of lengths a and b, associated with corresponding internal energies E_a and E_b. We will assume that there is no interaction energy between the molecules. Use the constant tension canonical ensemble to calculate the canonical partition function. Derive a formula for the length of the polymer as a function of the inverse temperature and tension.

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Part II. Section B. Thermodynamics and Statistical Mechanics (continued)

3. Consider a 3D gas of electrons in a large box of size L, under a uniform magnetic field B in the vertical direction. In this problem we will ignore the spin of the electrons.

a) What are the energy levels and their degeneracies?

b) Write the grand canonical partition sum, and compute the pressure as a function of the activity $z = e^{\beta\mu}$ and the inverse temperature $\beta = (kT)^{-1}$. Assume you are in the low density regime.

c) Find the magnetization and the magnetic susceptibility χ when $B = 0$, still at low density. Express your answers in terms of β and of the density ρ.

d) Does the system display ferromagnetism, diamagnetism, or paramagnetism? Explain your answer.