PHYSICS DEPARTMENT
PRINCETON UNIVERSITY

GRADUATE PRELIMINARY EXAMINATION

Thursday, May 3, 2001 — 9:00 am – 12:00 noon

Part I.

Answer two out of the three questions in Section A (Mechanics) and two out of the three questions in Section B (Electricity and Magnetism).

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

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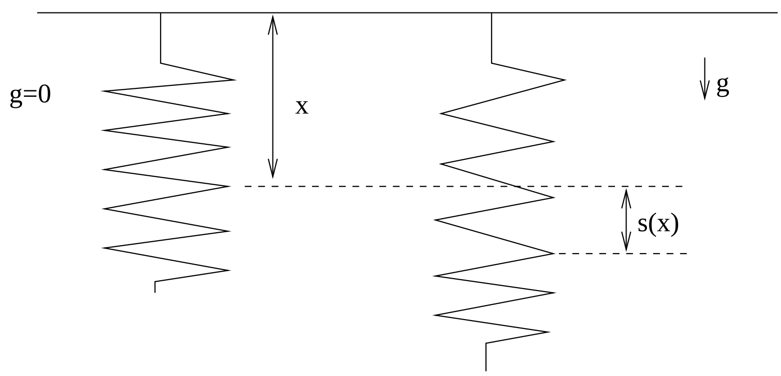
Part I. Section A. Mechanics

1. A spring has spring constant K, unstretched length L, and mass per unit length ρ. The spring is suspended vertically from one end in a constant gravitational field, g, and stretches under its own weight.



a) For a point whose distance from the upper end of the spring is x when unstretched, find its distance $s(x)$ from the upper end when the spring is stretched.

b) Suppose we suddenly 'turn off' gravity. (This can be done for example by putting the system in an elevator, which suddenly falls down freely from rest.) Find the subsequent motion $s(x, t)$ of the spring.

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Part I. Section A. Mechanics (continued)

2. A particle of mass m moves in a one-dimensional potential $V(x) = -ax^2 + bx^4$ with very light damping. The particle is set in motion with a large initial velocity. Suppose now we measure the period of the motion for each full oscillation, and call these periods T_1, T_2, T_3, T_4, and so on. It is observed that the T_i briefly become very large for i near some i_0.

   a) Explain what makes the periods get large.

   b) Obtain a scaling form for T_i near $i = i_0$, valid in the limit of small damping. (A scaling form would be something like $T \sim |i - i_0|^\alpha$ for some α, or $T \sim \log|i - i_0|$, etc). Hint: Consider first the motion without the friction, $m\ddot{x} = -V'(x)$. Recalling that this motion is necessarily periodic, derive an integral formula relating the period of oscillation to the energy and the turning points x_- and x_+ of the motion.

   c) Give an approximate sketch of T_i as a function of i.

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Part I. Section A. Mechanics (continued)

3. A particle of mass m and charge q moves freely in a gravitational field $\mathbf{g} = g\hat{\jmath}$ and a magnetic field $\mathbf{B} = B\hat{k}$. At time $t = 0$ the particle is released from the origin O with no initial velocity. It traces a curve in the x, y plane.

a) Find the parametric equations $x = x(t), y = y(t)$ describing the curve. Sketch the curve on an x, y diagram.

The above motion is idealized, because two effects have been ignored: air drag and radiation damping.

b) Now assume that the particle also feels a drag force due to the surrounding atmosphere, $F = -\beta v$. Derive the motion of the particle. What is its final velocity?

c) Instead of air drag, suppose we include the damping effect caused by the electromagnetic radiation emitted during its motion. Describe, qualitatively, how this modifies the motion found in part a). What is the final velocity of the particle?

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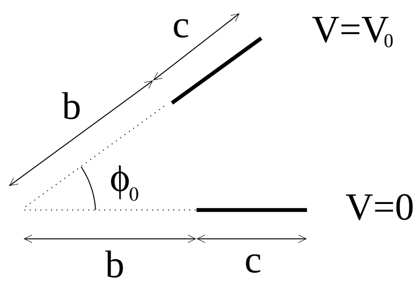
Part I. Section B. Electricity and Magnetism

1. Two identical plates of length c and width d are separated by an angular separation of ϕ_0 as shown. The plate at $\phi = 0$ is grounded, and the plate at $\phi = \phi_0$ is set at potential V_0.



a) Compute the stored energy in the capacitor. Assume that the electrical potential between the plates depends only on ϕ, and ignore fringe fields. (In which limit is this an allowed approximation?).

Now take ten plates in a cylindical arrangement, and connect them as follows:

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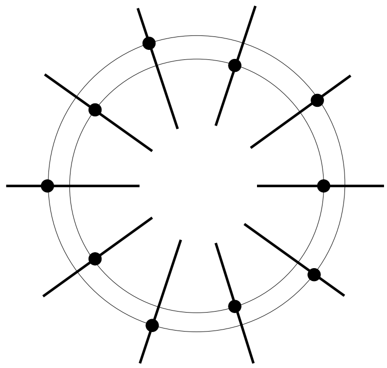
Part I. Section B. Electricity and Magnetism (continued)

Problem 1. (continued)



The odd plates are all connected together with a wire. The even plates are also all connected together. There is no direct connection between the odd and even plates. Assume a charge Q is placed on the even plates, and a charge $-Q$ on the odd plates.

b) Compute the total capacitance of this structure.

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Part I. Section B. Electricity and Magnetism (continued)

2. In this problem, we investigate the effect of electromagnetic waves traveling through a gas of charged particles. This can happen when there is radio emission from a pulsar, and these signals propagate through clouds of charged particles in deep space before being detected on Earth. A linearly polarized radio wave will induce a charged current in the cloud which is proportional to the time-dependent electric field of the plane wave (ignore the motion of the charged particles due to the magnetic field of the plane wave).

   a) Show that the dispersion relation between the frequency ω and the wavevector k for plane waves traveling through an electron gas can be written in terms of

$$1 - \frac{\omega_p^2}{\omega^2}$$

   where ω_p is the plasma frequency. Express the plasma frequency in terms of: $m_e = 9.1 \times 10^{-28}$ g (the mass of the electron), $-e = -4.8 \times 10^{-10}$ esu (the electron charge), and n_e (the volume density of electrons in the cloud).

   b) For radio wave frequencies above ω_p, how significant is the dispersion from ions (protons) in comparison to electrons?

   c) Evaluate the phase velocity ω/k and the group velocity $d\omega/dk$ and compare them to the speed of light. Write the phase and group velocities in terms of the ratio ω/ω_p.

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Part I. Section B. Electricity and Magnetism (continued)

Problem 2. (continued)

d) The Vela pulsar is about 500 parsecs distant (1 parsec= 3×10^{18} cm). It emits radio waves over a broad band. When observations are made in narrow frequency bands, what is observed are narrow pulses which arrive at a fixed period, similar to a timing signal for synchronizing a clock.

e) The narrow pulses observed at 1660 MHz are delayed relative to the narrow pulses observed at 1720 MHz by 6.8 ms. If this is interpreted by the dispersion in an ionized gas, what is the mean density of free electrons between Vela and us? To simplify the calculation, you can anticipate that $\omega_p \ll \omega$.

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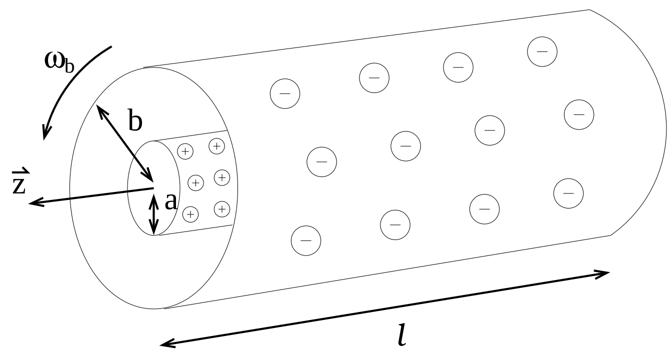
Part I. Section B. Electricity and Magnetism (continued)

3. Two long, thin concentric hollow cylindrical shells are each free to rotate around the z-axis. A mechanical attachment (not shown) keeps them concentric. The two cylinders have the same length ℓ, but different radii a and b. Each cylinder is an insulator, with a fixed charge per unit area, given by σ_a and σ_b, respectively



a) Initially, both cylinders are at rest. Compute the electric field inside, between and outside the cylinders. You can ignore the fringe fields at the ends of the cylinders.

b) What is the relation between σ_a and σ_b such that $\vec{E} = 0$ outside the outer cylinder?

c) Suppose that the inner cylinder is held at rest, while the outer cylinder rotates at angular frequency ω_b. Compute the magnetic field.

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Part I. Section B. Electricity and Magnetism (continued)

Problem 3. (continued)

From now on assume that σ_a and σ_b are related such that $\vec{E} = 0$ outside the outer cylinder.

d) At what frequency ω_a does the inner cylinder need to rotate such that $\vec{B} = 0$ inside of it?

The two cylinders are attached so that they rotate together $\omega = \omega_a = \omega_b$. The cylinders begin at rest and are driven with an external torque until they reach a final angular frequency ω. It is noticed that the induced magnetic flux through the cylinders causes a back emf which opposes their rotation.

e) Compute the additional external torque needed to overcome the back emf. (Hint: Use Faraday's law.)

f) Calculate the angular momentum in the electromagnetic field from the direct integration of the expression (given in MKS units)

$$\vec{L}_{\rm EM} = \epsilon_0 \int \vec{x} \times (\vec{E} \times \vec{B}) d^3x \quad .$$

Does this angular momentum correspond to the time integration of the torque computed in part e)?